TD n°3

Exercice-3 (Corrigé de la question 2)

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2) Déplacements de cet état de déformations avec les conditions sur les constantes a, b et c

$$\varepsilon_{x_1}=0,\varepsilon_{x_2}=0,\varepsilon_{x_3}=0,\varepsilon_{x_1x_2}=0,\varepsilon_{x_1x_3}=bx_2x_3,\varepsilon_{x_2x_3}=cx_1x_3$$

$$\varepsilon_{x_1}=\frac{\partial u_1}{\partial x_1}=0,\varepsilon_{x_2}=\frac{\partial u_2}{\partial x_2}=0,\varepsilon_{x_3}=\frac{\partial u_3}{\partial x_3}=0,\varepsilon_{x_1x_2}=\frac{1}{2}\left(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\right)=0$$

$$\varepsilon_{x_1x_3}=\frac{1}{2}\left(\frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}\right)=-cx_2x_3,\varepsilon_{x_2x_3}=\frac{1}{2}\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right)=cx_1x_3$$

$$u_1=F_1(x_2,x_3),u_2=F_2(x_1,x_3),u_3=F_3(x_1,x_2),\frac{\partial F_2(x_1,x_3)}{\partial x_1}+\frac{\partial F_1(x_2,x_3)}{\partial x_2}=0$$

$$\frac{\partial F_2(x_1,x_3)}{\partial x_1}=-\frac{\partial F_1(x_2,x_3)}{\partial x_2}=f(x_3),\quad F_1(x_2,x_3)=-f(x_3)x_2+h(x_3),\ F_2(x_1,x_3)=f(x_3)x_1+g(x_3)$$

$$u_1(x_1,x_2,x_3)=-f(x_3)x_2+h(x_3)\qquad u_2(x_1,x_2,x_3)=f(x_3)x_1+g(x_3)$$

$$\frac{\partial u_3(x_1,x_2,x_3)}{\partial x_1}-f'(x_3)x_2+h'(x_3)=-2cx_2x_3\quad \frac{\partial}{\partial x_1}F_3(x_1,x_2)=-h'(x_3)+x_2(f'(x_3)-2cx_3)$$

$$\begin{cases}f'(x_3)-2cx_3=cte=A_1\\ h'(x_3)=cte=A_2\\ \dfrac{\partial}{\partial x_1}F_3(x_1,x_2)=A_1x_2-A_2\end{cases}\qquad \begin{cases}f(x_3)=A_1x_3+cx_3^2+A_3\\ h(x_3)=A_2x_3+A_4\\ F_3(x_1,x_2)=A_1x_2x_1-A_2x_1+G(x_2)=u_3(x_1,x_2)\end{cases}$$

$$\frac{\partial u_3(x_1,x_2,x_3)}{\partial x_2}+f'(x_3)x_1+g'(x_3)=2cx_1x_3\qquad A_1x_1+G'(x_2)+(A_1+2cx_3)x_1+g'(x_3)-2cx_1x_3=0$$

$$A_1x_1+G'(x_2)+A_1x_1+g'(x_3)=0\quad \begin{cases}A_1=0\\ G'(x_2)=cte=A_5\\ g'(x_3)=cte=-A_5\end{cases}\quad \begin{cases}G(x_2)=A_5x_2+A_6\\ g(x_3)=cte=-A_5x_3+A_7\end{cases}$$

$$\begin{cases}u_1(x_1,x_2,x_3)=-(cx_3^2+A_3)x_2+A_2x_3+A_4\\ u_2(x_1,x_2,x_3)=(cx_3^2+A_3)x_1-A_5x_3+A_7\\ u_3(x_1,x_2,x_3)=-A_2x_1+A_5x_2+A_6\end{cases}\quad \begin{cases}u_1(x_1,x_2,x_3)=-cx_2x_3^2-A_3x_2+A_2x_3+A_4\\ u_2(x_1,x_2,x_3)=cx_1x_3^2+A_3x_1-A_5x_3+A_7\\ u_3(x_1,x_2,x_3)=-A_2x_1+A_5x_2+A_6\end{cases}$$

$$\begin{bmatrix}u_1(x_1,x_2,x_3)\\ u_2(x_1,x_2,x_3)\\ u_3(x_1,x_2,x_3)\end{bmatrix}=\begin{bmatrix}-cx_2x_3^2\\ cx_1x_3^2\\ 0\end{bmatrix}+\begin{bmatrix}-A_3x_2+A_2x_3\\ A_3x_1-A_5x_3\\ -A_2x_1+A_5x_2\end{bmatrix}+\begin{bmatrix}A_4\\ A_7\\ A_6\end{bmatrix}$$